Quiz 3: 14.1-14.3

Show all work clearly.

(1) Given $f(x,y) = x^2 y^3 + y\cos(xy) - \sqrt{4-x^2}$, find the first order partial derivatives f_x, f_y.

$$f_x(x,y) = 2xy^3 - y^2\sin(xy) + \frac{x}{\sqrt{4-x^2}}$$
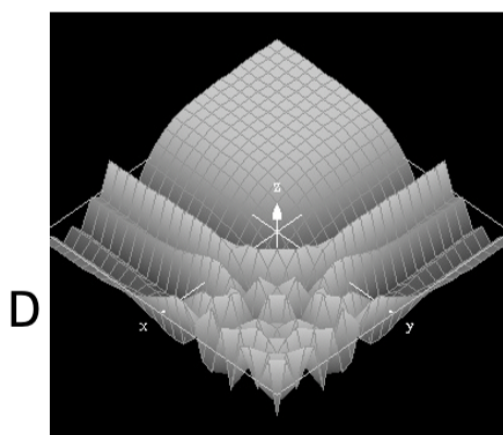
(8 points)

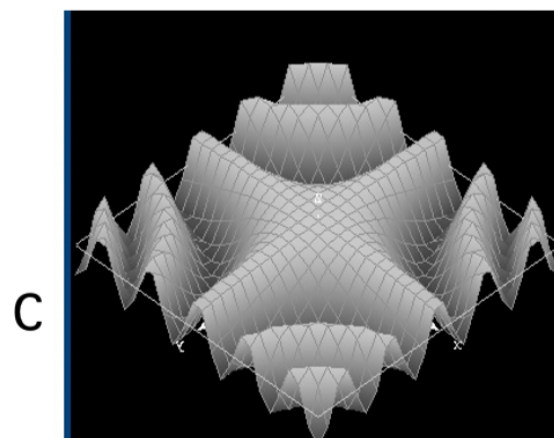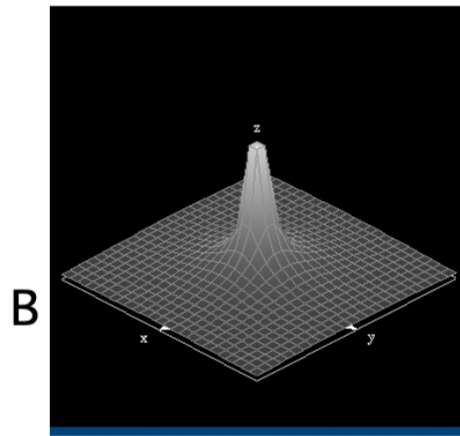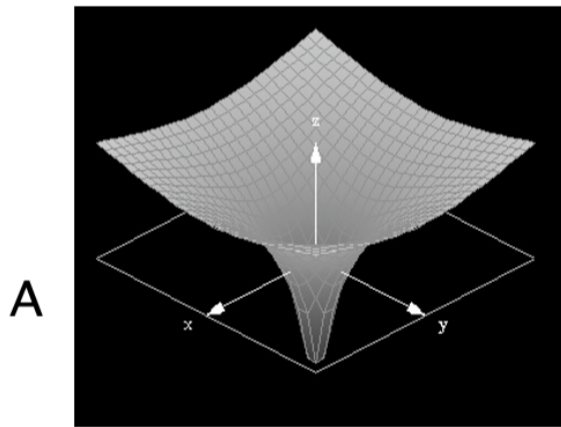$$f_y(x,y) = 3x^2y^2 + \cos(xy) - xy\sin(xy)$$

(2) Match the following equations to the corresponding graph. (4 points)

a) $f(x,y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ ___B___

(b) $f(x,y) = \cos(e^x + e^y)$ ___D___

c) $f(x,y) = \ln(x^2+y^2)$ ___A___

(d) $f(x,y) = \cos(xy)$ ___C___

A

B

C

D

(3) For the following graph, with the point (1,1,f(1,1)) plotted, determine if the following are positive or negative. (2 points)
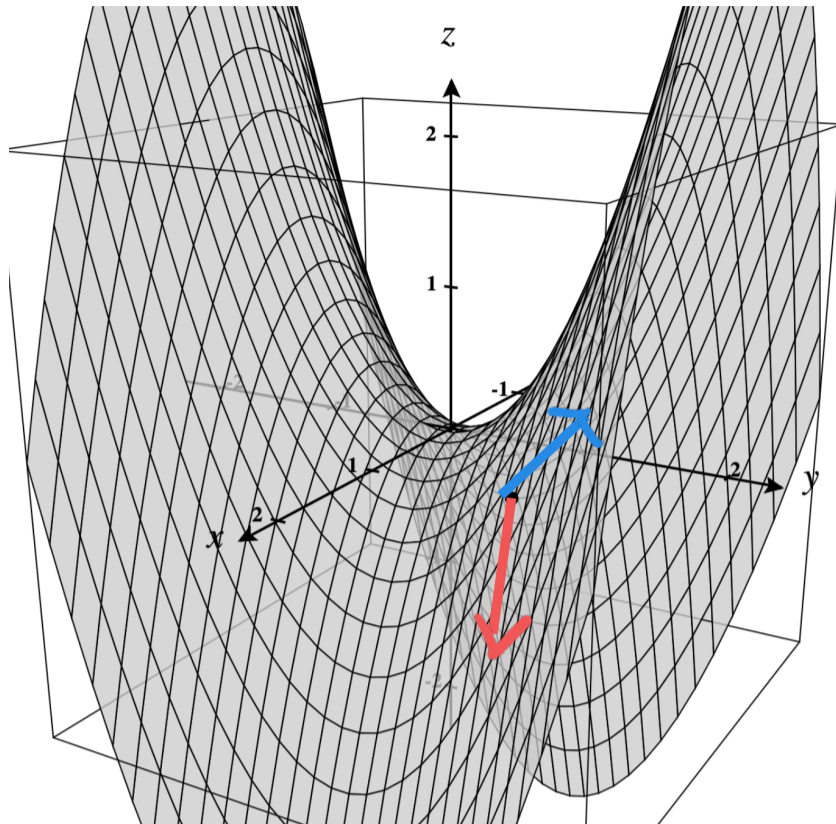
a) $f_x(1,1)$ _____

Explain

b) $\dfrac{\partial f}{\partial y}(1,1)$ _____

Explain

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(4) For the function $f(x,y) = \dfrac{x^3 y}{3x^6 + y^2}$ (SHOW WORK) (6 points)

(a) Find $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ along any straight line $y = mx$. _____

$$f(x, mx) = \frac{x^3(mx)}{3x^6 + (mx)^2} = \frac{mx^4}{3x^6 + m^2 x^2} = \frac{mx^2}{3x^4 + m^2}$$

$$\lim_{x\to 0} f(x, mx) = \frac{0}{m^2} = 0$$

(b) Find $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ along the curve $y = x^3$. _____ $\frac{1}{4}$

$$f(x, x^3) = \frac{x^3 x^3}{3x^6 + (x^3)^2} = \frac{x^6}{4x^6} = \frac{1}{4}$$

$$\lim_{x\to 0} f(x, y^3) = \frac{1}{4}$$

(c) What can be said about $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$? _____ DNE _____

Premember, if (a) and (b) were the same value, we can not draw any conclusion